A Marie-Curie-ITN
within H2020

Proceedings of the 1st MIGRATE Workshop
June 30-July 1st, 2016 – Strasbourg, France

MIGRATEWS2016-04

RAREFIED POISEUILLE GAS FLOW DUE TO HARMONICALLY OSCILLATING PRESSURE GRADIENT

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KEY WORDS

Rarefied gas dynamics, Oscillatory microchannel flow, BGK

ABSTRACT

The periodic Poiseuille flow due to an externally imposed harmonically oscillating pressure gradient is a classical problem in fluid mechanics and it is modelled based on the unsteady incompressible Navier-Stokes formulation under the assumption of fully developed flow. However, as it is pointed out in [1] this approach is valid under the provisions that both the mean free path and time are much smaller than the distance between the plates and the reference oscillating time respectively. If either of these restrictions is violated the problem must be tackled via kinetic theory.

In the present work the oscillatory Poiseuille flow at arbitrary oscillation frequency over the whole range of the Knudsen number is investigated. Assuming that the amplitude of the oscillation is small the problem is modelled by the linearized time-dependent Bhatnagar–Gross–Krook (BGK) kinetic equation subject to purely diffuse boundary conditions at the walls. This type of oscillatory flow may be applied in cooling, flow control, measuring devices, separators and other applications related to fluidic oscillators. Most of the available work on this topic is due to oscillatory boundary driven flows [1] and only few works exist with the oscillatory flow driven by pressure gradients varying sinusoidally with time [2].

Consider a monoatomic gas confined in an infinite long channel with the two plates located at $y' = \pm H'/2$ and the x-axis lying in the center along the plates. The gas is oscillating due to an imposed harmonically oscillating pressure gradient $P(t',x') = \mathbb{R}\left[P(x')\exp(-i\omega t')\right]$, where $\mathbb{R}$ denotes the real part of the complex expression and ω is the oscillation frequency, caused by a periodically moving membrane or piston. The flow is assumed to be harmonic in time, fully developed (independent of x) and varying in the y-direction. This oscillatory channel flow is characterized by two parameters: i) the rarefaction parameter $\delta = (P_0 H')/(\mu_0 \upsilon_0)$, where P_0 is a reference pressure, μ_0 is the gas viscosity at reference temperature T_0 and υ_0 is the most probable molecular velocity; ii) the oscillation parameter $\theta = P_0/(\mu_0 \omega)$. The flow is in the hydrodynamic regime provided that both $\delta \gg 1$ and $\theta \gg 1$.

To obtain a valid solution in the whole range of the gas rarefaction and for any oscillating frequency a kinetic theory approach is applied [1]. Furthermore, assuming that the amplitude of the oscillation is adequately small, linearization is introduced. Then, taking advantage of the one-dimensionality of the flow in the physical space, the main unknown is a complex perturbed

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distribution function of the form $\tilde{Y}(t',y',c_y)=\mathbb{R}\left[Y(y',c_y)\exp(-i\omega t')\right]$, where c_y is the y-component of the molecular velocity. Substituting this expression into the unsteady linearized BGK kinetic equation and following some routine manipulation it is deduced that $Y(y',c_y)$ obeys the complex integro-differential equation

$$(\theta-i)Y+c_y\frac{\partial Y}{\partial y}=\theta u_x-\frac{\theta}{2\delta}, \quad (1)$$

where $i^2=-1$, $-\delta/(2\theta)\le y\le\delta/(2\theta)$ and $u(y)=(1/\sqrt{\pi})\int Y(y,c_y)\exp(-c_y^2)dc_y$ is the bulk velocity. Since u is complex it may be written as $u=u_A(y)\exp\left[i\varphi_u(y)\right]$, where u_A and φ_u are the amplitude and the phase of the bulk velocity. The associated boundary conditions are $Y(\pm\delta/(2\theta),c_y)=0$ for $c_y \lessgtr 0$. In this flow configuration the main quantity of practical interest is the oscillatory complex flow rate, denoted as $G=G_A\exp[i\varphi_G]$, which is obtained by integrating the velocity field across the distance between the plates.

In Fig. 1, the flow rate amplitude and phase is plotted in terms of δ for indicative values of θ. With increasing δ, for small values of θ, the amplitude G_A is monotonically decreased, while for larger values of θ, it is initially reduced up to about $\delta=1$, then it is increased up to some δ depending on θ and finally it is reduced again. The phase φ_G is always negative and tends to zero as δ approaches the free molecular limit. It has been also confirmed, although not shown here, that for $\delta,\theta\gg 1$ the unsteady Navier-Stokes solution in the slip regime, as well as that as θ is increased the corresponding steady flow rate of the typical stationary Poiseuille flow, are recovered. These argument confirm the correctness of the solution in the transition regime.

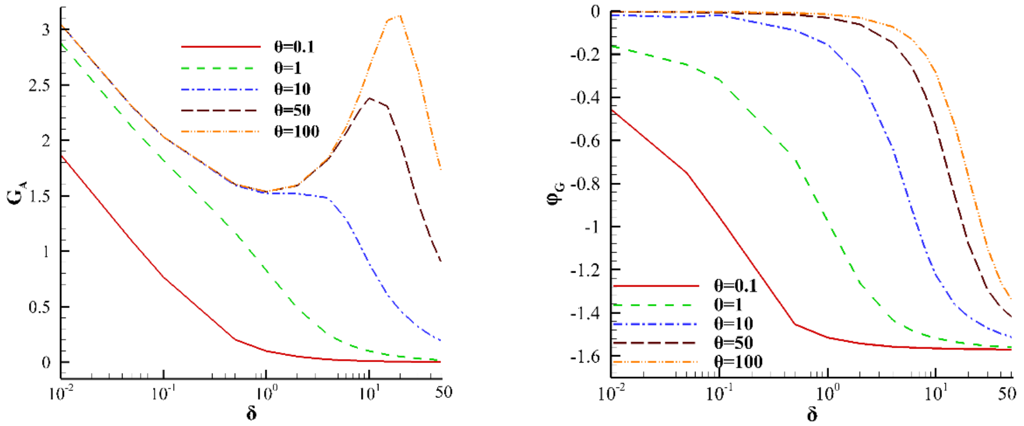

Figure 1: Flow rate amplitude and phase in terms of gas rarefaction and oscillation speed parameters

Acknowledgements

This work has been supported by the European Community under the contract of Association EURATOM/Hellenic Republic. The views and opinions expressed herein do not necessarily reflect those of the European Commission.

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