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A NEW METHOD FOR THE DETERMINATION OF MOMENTUM ACCOMMODATION COEFFICIENTS BY USING AXIAL BULK TEMPERATURE GRADIENTS

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ABSTRACT

The accurate determination of the momentum accommodation coefficient (α) between a dilute gas and the walls of a microchannel is essential for the determination of both velocity and temperature fields [1-4]. However, the measurement of this parameter is still an open question. Specific tests with the typical materials used in microfluidics are still needed.

In this work an original methodology for the determination of the momentum accommodation coefficient is theoretically derived by the analysis of the governing equations used for the study of the convective heat transfer in microchannels in presence of dilute gases.

For the analysis of convective heat transfer between the solid walls of a channel and a dilute gas the governing balance equations are generally coupled to the following boundary conditions:

$$\begin{cases} u_w - u_{wall} = \varpi_u \dfrac{2-\alpha}{\alpha} \lambda_{mfp} \left.\dfrac{\partial u}{\partial n}\right|_w \\ T_w - T_{wall} = \dfrac{2-\alpha_e}{\alpha_e} \dfrac{2\gamma}{\gamma+1} \dfrac{k}{\mu c_p} \lambda_{mfp} \left.\dfrac{\partial T}{\partial n}\right|_w \end{cases} \tag{1}$$

In Eq.(1) the momentum (α) and thermal (α_e) accommodation coefficients are invoked with γ, the specific heat ratio, and λ_{mfp}, the molecular mean free path. The coefficient ω_u is a corrective coefficient that must be added for a better prediction of the flow out of the Knudsen layer. By integrating the energy balance equation over the channel cross-section *Ω,* one obtains the following balance in terms of bulk temperature:

$$\rho c_p W \Omega \frac{dT_b}{dz} = q_{w,m}\Gamma + \mu \left[\int_\Gamma \left(u\nabla u \cdot (-\mathbf{n})\right) d\Gamma - \int_\Omega \left(u\nabla^2 u\right) d\Omega \right] + \mu \int_\Omega \left(u\nabla^2 u\right) d\Omega \tag{2}$$

Eq.(2) highlights that the increase in the bulk temperature along the channel axis is due to the heat flux imposed at the walls, to the viscous heating diminished by the cooling effect due to the flow work.

If one is interested to calculate the algebraic sum of the heat generated by viscous dissipation and the cooling effect due to the flow work term, it is easy to demonstrate that:

$$\mu\Delta = \mu \int_\Omega \left(\nabla u \cdot \nabla u\right) d\Omega - \int_\Omega u \frac{dp}{dz} d\Omega = \mu \int_\Gamma \left(u\nabla u\right) \cdot (-\mathbf{n}) d\Gamma \tag{3}$$

Eq.(3) highlights that, if a no-slip velocity boundary condition is valid on the channel solid walls, Δ becomes identically zero. This means that for a perfect gas the heat generated by viscous dissipation on the whole cross-section is exactly balanced by the cooling effect due to the flow work term. On the contrary, for a dilute gas in slip flow regime Δ is different from zero and assumes negative values; in this case Eq.(2) can be written in dimensionless form as:

$$\frac{d\theta_b}{dz^*} = 4\left(1 + \frac{Br\Delta^*}{\Gamma^*}\right) \qquad \text{with } \Delta^* = \int_{\Gamma^*} \left(u^* \nabla^* u^* \cdot (-\mathbf{n})\right) d\Gamma^* \tag{4}$$

It is easy to demonstrate that Δ^*, for a fixed Kn and for a fixed channel cross section geometry, is a monotonic function of the coefficient A_1:

$$A_1 = \varpi_u \frac{2-\alpha}{\alpha} \tag{5}$$

which depends to the corrective coefficient ω_u and the momentum accommodation coefficient (α).
This result suggests an innovative method for the determination of A_1 and α. In fact, Eq.(4) underlines that the value assumed by Δ^*, which depends on A_1, influences the axial bulk temperature variation along the fully developed region of the microchannel. From Eq.(4) the experimental determination of A_1 becomes possible by measuring the axial bulk temperature gradient in the thermally fully developed region of a microchannel heated at the walls with an uniform heat flux, if one knows exactly the cross-section geometry of the channel and the value of the Brinkman number. If the value of the Knudsen number is known, it is possible to calculate numerically the value of A_1 which allows to obtain exactly the same value of Δ^* deducted experimentally by Eq.(4).
However, this method can be applied only if the flow conditions through the microchannel are able to guarantee large Brinkman numbers (significant viscous dissipation effects) when the gas can be still considered as incompressible. It has been checked by using a model made with Comsol that the axial temperature gradient of a gas flowing in a microtube having an inner diameter of 10 μm under a heat flux of 1000 W/m^2 with an average velocity of 7.5 m/s and an inlet pressure equal to 0.7 bar can be used in order to test this procedure (Br=0.1).

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